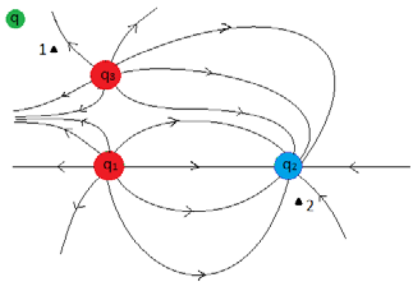
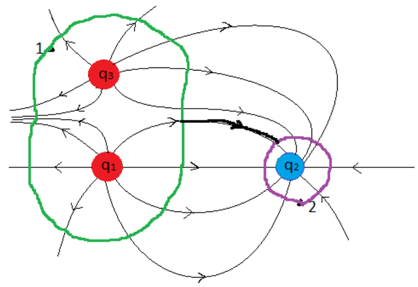
Homework 5 Solutions: Electric Potential Energy

*** You'll probably want to consider the results of homework 3 to expedite things here ☺**

Problem 1. Consider a green charge, q, sitting all by itself in the corner.





(a) Draw a path going through point 1 that q could travel along without depositing/extracting any energy into/from the ambient electric field.
This is an equipotential, drawn above.

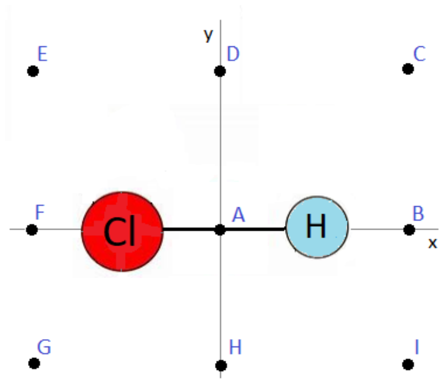
(b) Then do the same for point 2.
This is also an equipotential, also drawn above.

(c) Say q is positive. If you move q from point 1 to point 2 will you put energy into the electric field, or take energy out?
$\Delta PE_E = q\Delta V$. Now ΔV is negative, since black path goes with field lines. So ΔPE_E would be negative (energy coming out of field) if q is positive.

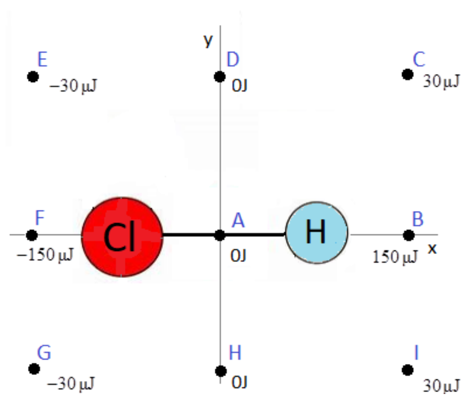
(d) Say q is negative. If you move q from point 1 to point 2, will you put energy into the electric field, or take energy out?
If q is negative, then ΔPE_E would be positive, so energy would be going into the field.

Problem 2. Does the electric force always point in the direction of most rapidly decreasing potential? How about most rapidly increasing potential? Or does it always point in the direction of most rapidly decreasing potential energy? Or increasing potential energy?
Always points in direction of most rapidly decreasing potential energy. So basically, the electric field is always trying to lower its energy, just like everyone else.

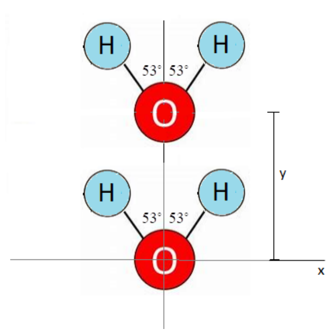
Problem 3. Say we take our green (5μC) charge from previous question, and move it from infinitely far away to each of the following points surrounding our HCl molecule. Label next to the points how much PE the electric field would gain or lose in each case.



Well, $PE_E = qV$, so,



Problem 4. Now let's go back to the water molecules. So when y was roughly 130pm, we found the electric force on the top O to be zero, and that this therefore was approximately where the top water molecule would want to snuggle in next to the bottom water molecule.



(a) Again, for simplicity, just considering the interaction of the bottom water molecule with the top O, would the electric field lower or increase its energy by having the top water molecule assume a position 130pm away from the bottom one?

Well, the potential generated by the bottom water molecule was:

$$V(y) = k(0.70e)\left[\frac{1}{\sqrt{(y-60\text{pm})^2+(80\text{pm})^2}} - \frac{1}{y}\right]$$

So at y = 130pm, the potential would be:

$$V(130\text{pm}) = k(0.70e)\left[\frac{1}{\sqrt{(130\text{pm}-60\text{pm})^2+(80\text{pm})^2}} - \frac{1}{130\text{pm}}\right] = 1.73\,\text{V}$$

The change in energy of the field associated with lowering the O atom into position would be:

$$\begin{aligned}\Delta PE_E &= q\Delta V \\ &= (-0.70e)(1.73\text{V}) \\ &= -1.94\times10^{-19}\,\text{J}\end{aligned}$$

So the electric field would lower its energy.

(b) How much energy would we have to put into a molecule (by raising their temperature, or whatever) to liberate it from its bond?

This would be the same: 1.94×10^{-19}J.

(c) Let's say we have a 1kg block of ice. And assume there is one such bond per molecule holding everything together. How much energy would we have to input to vaporize the ice? This is called the latent heat of vaporization. Compare to 2256kJ/kg (it's not a super-favorable comparison because of the blatant neglect of things like the other two H's on the top water molecule, but we're at least within a power of 10! Physicists are often content with such agreement when doing 'back of the envelope' calculations such as these)

So we just need to know how many molecues comprise 1kg of ice. And this is:

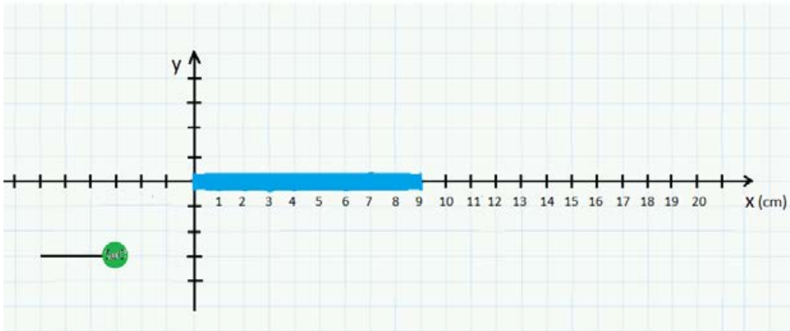
$$N = \frac{\text{mass}}{\text{m}_{molar}} \times N_A = \frac{1\,\text{kg}}{0.018\,\text{kg}} \times 6.022\times10^{23} = 3.35\times10^{25}$$

And then we multiply this number by the bond energy:

$$L_v = N\cdot PE_{E,bond} = (3.35\times10^{25})(1.94\times10^{-19}) = 6490\,\text{kJ}$$

So we're in the ballpark, though overestimating by a factor of 3 or so, and partly this is due to neglect of the presence of the H's which would serve to reduce the PE in these bonds.

Problem 5. Now consider that plastic rod from before, lying on the x-axis between 0 and 9cm, and charged uniformly to 10nC. Our 5μC charge is placed on a wire at coordinate (-3cm, -3cm), along which it can slide, in the 3rd quadrant. When released, how fast will it be going when it reaches the end of the wire? You can take its mass to be 12μg.



This is just conservation of energy. Refering to HW 3, where we got the potential generated by the rod:

$$V(x,y)=1000\ln\left[\frac{\sqrt{(x-0.09)^2+y^2}+0.09-x}{\sqrt{x^2+y^2}-x}\right]$$

We have:

$$0=\Delta PE_E+\Delta KE$$

$$-q\Delta V=\frac{1}{2}mv^2$$

$$-(5\times10^{-6})\left[V(-0.06,-0.03)-V(-0.03,-0.03)\right]=\frac{1}{2}(12\times10^{-9})v^2 \qquad (\mu\text{g}=10^{-6}\text{g}=10^{-9}\text{ kg})$$

$$-(5\times10^{-6})\left[867-1210\right]=\frac{1}{2}(12\times10^{-9})v^2$$

$$v=\sqrt{\frac{2(5\times10^{-6})(1210-867)}{12\times10^{-9}}}=535\,\text{m/s}$$

Problem 6. Let's go back to the ominous 3m radius cloud of electric dust, with density ρ = 0.05kg/m³. Its field was given by this below. (a) How much electric potential energy is stored in this cloud? (b) If a spark should precipitate conversion of all this PE_E into KE, how fast would the dust particles' average speed be (it's not *that* big, but you *would* get a ticket for driving this fast, at least in Florida).

$$E=\frac{kq_{enclosed}}{r^2}=\begin{cases}1.33\times10^6 r & \text{inside}\\ \dfrac{36\times10^6}{r^2} & \text{outside}\end{cases}$$

(a) So potential energy is given by:

$$
\begin{aligned}
PE_E &= \int \frac{\varepsilon_0}{2} E^2 dV \\
&= \int_0^3 \frac{\varepsilon_0}{2} (1.33\times 10^6 r)^2 \cdot 4\pi r^2 dr + \int_3^\infty \frac{\varepsilon_0}{2} \left(\frac{36\times 10^6}{r^2} \right)^2 \cdot 4\pi r^2 dr \\
&= \int_0^3 98 r^4 dr + \int_3^\infty \frac{72000}{r^4} dr \\
&= 98 \cdot \frac{r^5}{5} \bigg|_{r=0}^{r=3} - \frac{72000}{r^3} \bigg|_{r=0}^{r=3} \\
&= 7.4\times 10^3\,\mathrm{J} = 7.4\,\mathrm{kJ}
\end{aligned}
$$

(b) So conversion of all this energy into KE would result in an average velocity of:

$$PE_E = KE$$

$$7400 = \frac{1}{2} m v^2 \qquad m = \rho V = (0.05)\frac{4}{3}\pi (3)^3 = 5.6\,\mathrm{kg}$$

$$v = \sqrt{\frac{2(7400)}{5.6}} = 51\,\mathrm{m/s}$$